Warm-Up

CST/CAHSEE: Algebra II (Standard 24)	Review: Calculus (Standard 4.0)

Given that $f(x) = 3x^2 - 4$

and $g(x) = 2x - 6$ what is $g(f(2))$?

A -2
B 6
C 8
D 10

Given:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Find $f'(x)$ when $f(x) = 3x^2 - 5$

Current: Calculus (Standard 4.0)	Other: Calculus (Standard 4.0)

Power Rule for Derivatives

Given:
$$f(x) = x^n$$
$$f'(x) = nx^{n-1}$$

Use the power rule to find the derivative.

$$f(x) = 3x^2 - 5$$

Use the power rule to find the derivative of

$$f(x) = 5x^3 - 7x^2 + 8x - 11$$

Derivative Theorems

Standards:
Algebra 1 2.0, 4.0, 6.0, 7.0, 11.0, 16.0, 17.0, 18.0
Calculus 4.0

Objectives:
Use Derivative Theorems to find Derivatives
Connect Algebra standards to Calculus

Debrief Warm-Up and quickly review definition of derivative and notations for derivative.

(solutions to the Warm-Up are worked out at the end of the lesson)

$f'(x),\ y',\ \frac{dy}{dx},\ \frac{d}{dx}$, slope of the tangent line to a curve, instantaneous rate of change

Distribute the handout titled, "Derivative Theorems" to the class.

(you will find this handout at the end of the lesson)

Example 1: Find the derivative of the function $y=\left(2x^3+3\right)\left(x^4-2x\right)$ using two methods.

Method 1: Rewrite the function first, then use the Power Rule for Derivatives.	Method 2: Use the Product Rule for Derivatives
$y=\left(2x^3+3\right)\left(x^4-2x\right)$ $y=2x^3\left(x^4-2x\right)+3\left(x^4-2x\right)$ $y=2x^7-4x^4+3x^4-6x$ $y=2x^7-x^4-6x$ $\frac{dy}{dx}=14x^6-4x^3-6$	$y=\left(2x^3+3\right)\left(x^4-2x\right)$ $\frac{dy}{dx}=\left(2x^3+3\right)\left(x^4-2x\right)'+\left(x^4-2x\right)\left(2x^3+3\right)'$ $\frac{dy}{dx}=\left(2x^3+3\right)\left(4x^3-2\right)+\left(x^4-2x\right)\left(6x^2\right)$ $\frac{dy}{dx}=\left(8x^6-4x^3+12x^3-6\right)+\left(6x^6-12x^3\right)$ $\frac{dy}{dx}=14x^6-4x^3-6$

You Try!

Find the derivative of the function $y=(6x+5)(x^3-2)$ using two methods.

Method 1: Rewrite the function first, then use the Power Rule for Derivatives.	Method 2: Use the Product Rule for Derivatives
$y=(6x+5)(x^3-2)$ $y=6x(x^3-2)+5(x^3-2)$ $y=6x^4-12x+5x^3-10$ $y=6x^4+5x^3-12x-10$ $y'=24x^3+15x^2-12$	$y=(6x+5)(x^3-2)$ $y'=(6x+5)(x^3-2)'+(x^3-2)(6x+5)'$ $y'=(6x+5)(3x^2)+(x^3-2)(6)$ $y'=18x^3+15x^2+6x^3-12$ $y'=24x^3+15x^2-12$

Example 2: Find the derivative of the function $f(x)=\frac{3x-1}{2x+1}$ using the Quotient Rule for Derivatives.

$$f(x)=\frac{3x-1}{2x+1}$$

$$f'(x)=\frac{(2x+1)(3x-1)'-(3x-1)(2x+1)'}{(2x+1)^2}$$

$$f'(x)=\frac{(2x+1)(3)-(3x-1)(2)}{(2x+1)^2}$$

$$f'(x)=\frac{6x+3-(6x-2)}{(2x+1)^2}$$

$$f'(x)=\frac{6x+3-6x+2}{(2x+1)^2}$$

$$f'(x)=\frac{5}{(2x+1)^2}$$

The derivatives of the six trigonometric functions are given at the bottom of the Derivative Theorems handout. We will now take a look at the derivative of the tangent function.

Example 3: Use the Quotient Rule for Derivatives to show that $\frac{d}{dx}\tan x = \sec^2 x$.

$$\begin{aligned}
\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)\\
&= \frac{\cos x(\sin x)' - \sin x(\cos x)'}{\cos^2 x}\\
&= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}\\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}\\
&= \frac{1}{\cos^2 x} \qquad \left(\text{since } \cos^2 x + \sin^2 x = 1\right)\\
&= \sec^2 x \qquad \left(\text{since } \frac{1}{\cos x} = \sec x\right)\\
\therefore\ \frac{d}{dx}\tan x &= \sec^2 x
\end{aligned}$$

You Try!

Use the Quotient Rule for Derivatives to show that $\frac{d}{dx}\cot x = -\csc^2 x$.

$$\begin{aligned}
\frac{d}{dx}\cot x &= \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)\\
&= \frac{\sin x(\cos x)' - \cos x(\sin x)'}{\sin^2 x}\\
&= \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}\\
&= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}\\
&= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}\\
&= \frac{-1}{\sin^2 x} \qquad \left(\text{since } \cos^2 x + \sin^2 x = 1\right)\\
&= -\csc^2 x \qquad \left(\text{since } \frac{1}{\sin x} = \csc x\right)\\
\therefore\ \frac{d}{dx}\cot x &= -\csc^2 x
\end{aligned}$$

Note: The derivatives for secant and cosecant can be found similarly by using the Quotient Rule.

You Try!

Find the derivative of the function $y = \dfrac{\sqrt{x}}{\cos x}$.

Method 1: Using Radical Notation	Method 2: Using Fractional Exponent Notation
$y = \dfrac{\sqrt{x}}{\cos x}$	$y = \dfrac{\sqrt{x}}{\cos x}$
$\dfrac{dy}{dx} = \dfrac{\cos x\left(\sqrt{x}\right)' - \sqrt{x}\left(\cos x\right)'}{\cos^2 x}$	$y = \dfrac{x^{\frac{1}{2}}}{\cos x}$
$\dfrac{dy}{dx} = \dfrac{\cos x\left(\dfrac{1}{2\sqrt{x}}\right) - \sqrt{x}\left(-\sin x\right)}{\cos^2 x}$	$\dfrac{dy}{dx} = \dfrac{\cos x\left(x^{\frac{1}{2}}\right)' - x^{\frac{1}{2}}\left(\cos x\right)'}{\cos^2 x}$
$\dfrac{dy}{dx} = \dfrac{\dfrac{\cos x}{2\sqrt{x}} + \sqrt{x}\sin x}{\cos^2 x}$	$\dfrac{dy}{dx} = \dfrac{\dfrac{1}{2}x^{-\frac{1}{2}}\cos x - x^{\frac{1}{2}}\left(-\sin x\right)}{\cos^2 x}$
$\dfrac{dy}{dx} = \dfrac{\dfrac{\cos x}{2\sqrt{x}} + \sqrt{x}\sin x}{\cos^2 x} \bullet \dfrac{2\sqrt{x}}{2\sqrt{x}}$	$\dfrac{dy}{dx} = \dfrac{\dfrac{1}{2}x^{-\frac{1}{2}}\cos x - x^{\frac{1}{2}}\left(-\sin x\right)}{\cos^2 x} \bullet \dfrac{2}{2}$
$\dfrac{dy}{dx} = \dfrac{\cos x + 2x\sin x}{2\sqrt{x}\cos^2 x}$	$\dfrac{dy}{dx} = \dfrac{x^{-\frac{1}{2}}\cos x + 2x^{\frac{1}{2}}\sin x}{2\cos^2 x}$
	$\dfrac{dy}{dx} = \dfrac{x^{-\frac{1}{2}}\left(\cos x + 2x\sin x\right)}{2\cos^2 x}$
	$\dfrac{dy}{dx} = \dfrac{\cos x + 2x\sin x}{2x^{\frac{1}{2}}\cos^2 x}$
	$\dfrac{dy}{dx} = \dfrac{\cos x + 2x\sin x}{2\sqrt{x}\cos^2 x}$

Distribute handout "Derivative Multiple-Choice Practice – Released Advanced Placement Exam Questions" to the class for more practice ☺

(you will find the handout at the end of the lesson)

End of Lesson – Handouts on the following pages

Solutions to Warm-Up

1)	2)
$g(f(2)) = 2\left(3(2)^2 - 4\right) - 6$ $= 2\left(3(4) - 4\right) - 6$ $= 2(12 - 4) - 6$ $= 2(8) - 6$ $= 16 - 6$ $= 10$ D	$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$ $f'(x) = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 5 - 3x^2 + 5}{h}$ $f'(x) = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$ $f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2}{h}$ $f'(x) = \lim_{h \to 0} \frac{3h(2x+h)}{h}$ $f'(x) = \lim_{h \to 0} \left[3(2x+h)\right]$ $f'(x) = 3(2x+0)$ $f'(x) = 6x$
3)	4)
$f(x) = 3x^2 - 5$ $f'(x) = 6x$	$f(x) = 5x^3 - 7x^2 + 8x - 11$ $f'(x) = 15x^2 - 14x + 8$

Derivative Theorems

Power Rule

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Sum Rule

If *f* and *g* are differentiable at *x*, then

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

Constant Multiple Rule

If *f* is differentiable at *x*, then for any constant c,

$$\frac{d}{dx} c \bullet f(x) = c \bullet f'(x)$$

Product Rule

If *f* and *g* are differentiable at *x*, then

$$\frac{d}{dx}(fg)(x) = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule

If *f* and *g* are differentiable at *x* and $g(x) \neq 0$, then

$$\frac{d}{dx}\left(\frac{f}{g}\right)(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Chain Rule (for composition of functions)

If *g* is differentiable at *x* and *f* is differentiable at $g(x) \neq 0$, then $f \circ g$ is differentiable at *x* and

If $h(x) = f(g(x))$

then $h'(x) = f'(g(x)) \bullet g'(x)$

or, If $y = f(u)$ where *u* is a differentiable function of *x*, then,

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

Derivative Multiple-Choice Practice – Released Advanced Placement Exam Questions

Make an answer column for your multiple-choice answers. No calculators.

1) Find $f'(x)$: $f(x) = 5x^4 - 6x^3 + 14x - 7$

[A] $f'(x) = 5x^3 - 6x^2 + 14$ [B] $f'(x) = 20x^3 - 18x^2 + 14$

[C] $f'(x) = 20x^4 - 18x^3 + 14x - 7$ [D] $f'(x) = 5x^3 - 3x^2 + 14$

[E] None of these

2) If $f(3) = 4$ and $f'(3) = -2$, find the equation of the tangent line when $x = 3$.

[A] $y + 4 = -2(x - 3)$ [B] $y + 4 = -2(x + 3)$

[C] $y - 3 = -2(x - 4)$ [D] $y - 4 = -2(x - 3)$

[E] None of these

3) Let $f(-1) = 5$, $f'(-1) = 2$, $g(-1) = -6$, $g'(-1) = 3$. Find $h'(-1)$ if $h(x) = \dfrac{f(x)}{g(x)}$.

[A] $-\dfrac{1}{2}$ [B] $-\dfrac{3}{4}$ [C] $\dfrac{1}{2}$

[D] $\dfrac{3}{4}$ [E] None of these

4) If $f(x) = \sec x$, which of the following is equivalent to $f'(x)$?

[A] $1 + \tan^2 x$ [B] $\lim\limits_{h \to 0} \dfrac{\sec(x+h)\tan(x+h) - \sec x \tan x}{h}$

[C] $\csc x$ [D] $\lim\limits_{h \to 0} \dfrac{\sec(x+h) - \sec x}{h}$ [E] None of these

5) Find the instantaneous rate of change of w with respect to x if $w = \dfrac{8}{5x^4}$.

[A] $\dfrac{8}{20x^3}$ [B] $-\dfrac{32}{5x^5}$

[C] $-\dfrac{32}{5x^3}$ [D] $-\dfrac{8}{20x^5}$ [E] None of these

6) Find $f'(x)$: $f(x) = \dfrac{x^2 - 6x}{\sqrt{x}}$.

[A] $\dfrac{3x^2 - 6x}{x}$ [B] $\dfrac{3x^2 + 6x}{2x\sqrt{x}}$

[C] $\dfrac{3}{2}x^{\frac{1}{2}} - \dfrac{3}{x^{\frac{1}{2}}}$ [D] $x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$ [E] None of these

Answers: 1) B 2) D 3) B 4) D 5) B 6) C